Heart Rate Estimation Algorithm From Wrist-based Photoplethysmogram Using Subspace Learning Method

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***Abstract*—Wrist based devices, like smart-watches, fitness bands and health monitors all provide a common sensor called Photoplethysmography (PPG) to measure optical pulse signal. This is usually used to derive the instantaneous heart-rate (HR), which is useful while doing any exercise or to monitor on a regular basis for chronic patients. However, one major issue with the signal is that it is easily corrupted by ambulatory motion generated by hand movements of the subject. Since, these devices also come equipped with an independent motion sensor, namely a tri-axes accelerometer, researchers have taken interest in trying to correct the motion artifact in PPG using the accelerometer as a reference noise signal. However, it is not a trivial problem and hence, even after a substantial body of research, the problem remains unsolved, especially when considering on-premise estimation due to the resource-constrained nature of wearable devices. In this paper, we aim to solve this problem using subspace based learning approach. Though this approach has been utilized before, we have added some novel steps to the algorithm pipeline and also made modifications so that the algorithm can be possibly run on a typical wearable device. Our preliminary results show efficacy and promise of our proposed approach.**

***Index Terms*—Photoplethysmography (PPG), Motion Artifacts, Heart Rate, PCA, Subspace Based Decomposition**

I. Introduction

With the emerging era of wearable technologies and smartwatches, one active area of research that these devices have been used for, is longitudinal monitoring of physiological signals. The most pervasive physiological sensor available on wrist wearables is the Photoplethysmogram or PPG sensor. However, this optical way of measuring arterial pulse has a major drawback in being susceptible to motion artifacts due to ambulation. Hence, discarding the motion artifacts during the estimation of physiological parameters like heart-rate and HRV (heart-rate variability) from ambulatory PPG becomes an important research problem.

In recent years, a number of research works have been proposed to enhance the signal quality of PPG in the presence of motion artifacts. Essentially, heart rate estimation algorithm could be characterized with three processing steps, namely, pre-processing, signal de-noising and post-processing. Pre-processing step involves baseline removal and basic filtering; signal de-noising includes further noise cleaning; post-processing part integrates heart rate tracking and smoothing. So far, various methods such as adaptive filtering [7] independent component analysis (ICA) [4], Kalman filtering [5], wavelet de-noising [6] have been investigated as effective de-noising methods. Recently, a generalized framework TROIKA [1] and JOSS [2] had been proposed which are quite effective and caters better accuracy on the data-set of 2015 IEEE Signal Processing Cup (SP Cup) competition [10]. However, since these algorithms integrate several parameters tuned for the data-set and heavily dependent on the post-processing method, their generalization capability is limited. Moreover, the computational load is another potential drawback while deploying the algorithm in resource-constrained wearable hardware.

Apart from these methods, recently machine learning based approaches [8], [9] have also gained popularity to remove motion artifacts from PPG signal. The rationale is to distinguish the right cardiac peak among the candidate peaks in the spectrum, based on some computed features. In practice, the realization of the machine learning based approach is quite strenuous owing to large training phase.

Acknowledging these issues, a robust heart rate estimation algorithm is proposed, which is characterized by the following features:

- Instead of directly working on the signal, we transform the signal into principal basis. Primarily, the PPG and the simultaneously acquired accelerometer signal are discretized and approximated by the number of principal components.

- To discard the principal components associated with motion or any other noise, two-stage verification method is employed. Firstly, according to the energy contribution, major principal components are only considered for signal reconstruction. Secondly, we have exploited the dominant frequency of principal component as a similarity metric between PPG and accelerometer.

- Eventually, the principal components allied with noise are discarded and the true cardiac signal is reconstructed for heart rate estimation.

II. Research Overview

Any physiological sensing by the wearable bio-sensor gets distorted when it is subjected to the physical motion. The conventional filtering method is reliably effective when the frequency range of the motion is not overlapped with the true heart rate signal. Incidentally, the human movement is very low-frequency signal and it coincides with actual heart rate signal(0.75 Hz to 3 Hz). Thus when the strong motion signal overlaps with the signal of interest, estimating the heart rate in time or frequency domain becomes a challenging problem. Evidently, the motion artifact is additive in nature and the acquired signal is an aggregate of both the true heart signal and the motion signal. Let's assume the following model represents the PPG signal perceived from the wrist:

$$P = H + M \tag{1}$$

Where $P \in \mathbb{R}^{N\times 1}$, is the received PPG signal. H is the true heart rate signal and M is the motion signal.

Since the significant portion of motion artifacts is not directly correlated to the actual PPG signal, we have aligned this problem to subspace learning method where two major components, motion signal and the true heart rate signal lies in two distinct subspaces. The main objective is to distinguish the motion signal subspace and eventually recovers the signal of interest. Principal Component Analysis(PCA) is employed as a subspace learning method which transforms the original time series signal into principal subspaces. Generally PCA is utilized as a dimensionality reduction technique; however, it could be used as a de-noising method by suitable selection of principal components. Essentially the PCA approximates the original time series signal into a number of constitutive principal components:

$$\mathbf{X} \approx \sum_{k=1}^{N} \mathbf{PC(k)} \tag{2}$$

Where $\mathbf{X}$ is the original time series signal.

III. Methodology

Figure 1 illustrates the complete methodology for Heart Rate Estimation algorithm.

A. *Preprocessing*

Preprocessing steps include baseline removal, the conventional filtering process and the normalization. The frequency range of Cardiac Signal spans from 0.75Hz to maximum 3Hz, which encompasses the heart beats per minute (BPM) from 42 BPM to 180 BPM. A bandpass filter with the same frequency range is applied to the raw PPG signal to discern the cardiac signal. The filtering process subsequently eliminates the sensor noise or any other noise outside of the signal of interest. Furthermore, the signal is normalized as the PCA based subspace learning based method is employed for subspace decomposition. The normalization process is defined as:

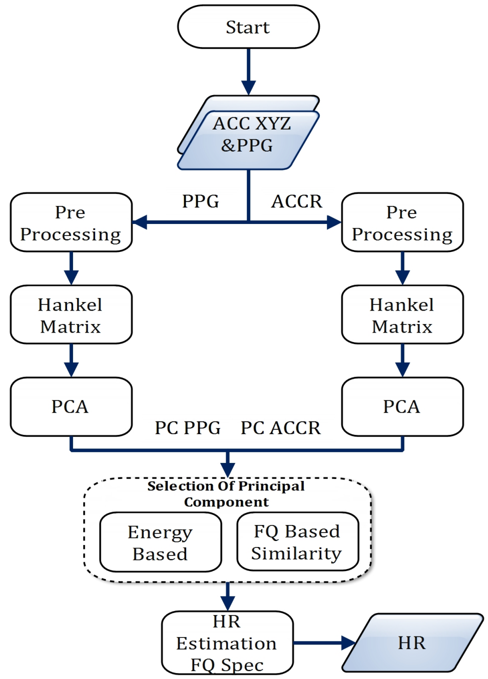


Fig. 1. Flow Chart of the proposed algorithm

$$\mathbf{Sig_{Norm}} = \frac{\mathbf{RawSig} - \mu}{\sigma} \tag{3}$$

Where μ is mean of the signal window and σ is the standard deviation.

B. *Subspace Decomposition*

1) Hankel Matrix Conversion: To accomplish the subspace decomposition, the original time series is mapped into a sequence of lagged vectors. Consider a time series data $X = \{x_1, x_2, ..., x_N\}$ where N is the number of total samples; is transformed into L lagged vectors. The L is called as the window length and for a meaningful interpretation, L must be chosen as $L < N/2$. The Trajectory matrix $TX \in R^{L\times K}$ of the time series X is formed where $K = N - L + 1$.

$$X \Longrightarrow TX_{i,j} = \begin{bmatrix} x_1 & x_2 & \cdots & x_L \\ x_2 & x_3 & \cdots & x_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_K & x_{K+1} & \cdots & x_N \end{bmatrix}$$

This trajectory matrix exhibits two important properties: 1) The diagonal of the matrix imparts the complete time series; moreover, rows and columns are the subseries of the actual time series. 2) Cross-diagonals of TX is $x_{j+i-1} = x_{i+j-1}$; thus it is called as Hankel Matrix.

2) Principal Component Analysis: Given a Hankel matrix $H \in \mathbb{R}^{m \times n}$, PCA aims to learn a projection matrix $W \in R^{n \times n}$, projecting the input data into n-dimensional subspaces. Let Y is the projected matrix and it is denoted as

$$\mathbf{Y} = \mathbf{HW} \tag{4}$$

Where columns of the projection matrix W represents the eigenvector computed from the covariance matrix of $\mathbf{HH^{\intercal}}$. The covariance matrix is defined as

$$\mathbf{C_H} = \frac{1}{\mathbf{N-1}}\mathbf{HH^{\intercal}} \tag{5}$$

Where N is the total number of samples.

The eigenvectors are structured in ascending order according to the eigenvalues and the leading eigenvector is the last column of the projection matrix W. The eigenvectors (EOFs) of the matrix $\mathbf{C_H}$ exploits the temporal covariance of the time series, computed at different lags and represented as Hankel matrix form. Essentially, the matrix Y is a projection of time series onto the Eigenvectors. The columns of the projected matrix Y are called principal components. These principal components are again time series of the same length of the Hankel matrix.

C. Subspace Decomposition of Accelerometer Signal

The motion subspace is approximated by the accelerometer signal. An acceleration sensor measures the acceleration in three axes; $\mathbf{ACC} \in \mathbb{R}^3$, however as we are not interested in direction of the motion the resultant is computed and considered for further processing. After preprocessing as described in section III, the subspace learning method is implied. Eventually, the resultant time series is approximated by the number of principal components:

$$\mathbf{ACC_R} \approx \sum_{k=1}^{N} \mathbf{PC_{ACC_R}(k)} \tag{6}$$

Where, $\mathbf{ACC_R}$ is the original time series accelerometer resultant signal and $\mathbf{PC_{ACC_R}(k)}$ is the PC vector.

D. Selection Of Principal Components

Having decomposed the PPG signal into orthogonal principal components, now we need to discard the principal components which are contributed by physical movement or any other noise rather than the true cardiac cycles. To recognize the components associated with motion or noise, two-stage verification mechanism are employed.

1) Energy based principal component selection: Based on the empirical analysis, it is observed that energy contributed by the principal component is pivotal for ascertaining the right principal components. The energy is estimated from the eigenvalues obtained from the eigendecomposition of the covariance matrix. It is defined as:

Algorithm 1: MARemovalSubspaceDecomposition

```
Data: PPG ∈ ℝ^{n×1} PPGDataWindow, ACCR ∈ ℝ^{n×1} ACCresultantDataWindow , WL
Result: CleanPPGV ∈ ℝ^{n×1}
begin
    Subspace Decomposition Using PCA.
      [PPGPC, PPGDiag, PEigenVal] ⟵
      SubspaceDecomposition(PPG, ACC, WL);
    [ACCPC, ACCDiag, AEigenVal] ⟵
      SubspaceDecomposition(PPG, ACC, WL);
    FQTHR ⟵ SamplingRate / FFTR;
    SizeV ⟵ (length(PPG) − WL + 1);
    s ⟵ WL;
    TotalEnP ⟵ 0;
    while true do
        PPGPCEn ⟵
          Energy(PPGPC, PEigenVal, PPGDiag, s)
        TotalEnP ⟵ (TotalEnP + PPGPCEn)
        MaxFQP ⟵ FTSpec(PPGPCV);
        n ⟵ WL;
        TotalEnA ⟵ 0;
        DiscardFlag ⟵ 0;
        while true do
            ACCPCEn ⟵
              Energy(ACCPCV, AEigenVal, ACCDiag, n)
            EnA ⟵ (EnA + ACCPCEn);
            MaxFQA ⟵ FTSpec(ACCPCV);
            if (((MaxFQP − MaxFQA) <=
              FQTHR)and(PPGPCEn <
              EnergyTHR)) then
                DiscardFlag ⟵ 1;
                break;
            if ((TotalEnA >= MAXEnPerThr) then
                break;
            n ⟵ n − 2;
        if ((DiscardFlag == 0) then
            CleanPPGV ⟵
              CleanPPGV + PPGPCV;
        if ((TotalEnP >= MAXEnPerThr) then
            break;
        s ⟵ s − 2;
```

$$\mathbf{Energy_{PC}} = \frac{\mathbf{EigenVal_{PC}}}{\mathbf{sum(EigenValDiag)}} \times 100 \tag{7}$$

Where $\mathbf{EigenValDiag} \in \mathbb{R}^{n \times 1}$ is a diagonal vector of Eigen value matrix obtained from Eigen decomposition.

In order to remove the unwanted noise, the major principal components are only considered where 90% of the energy is concentrated.

$$\mathbf{X} \approx \sum_{k=1}^{M} \mathbf{PC(k)} \tag{8}$$

Where $\mathbf{X}$ is the original PPG signal and $\mathbf{M} << \mathbf{N}$. The value of $\mathbf{M}$ is decided dynamically, according to the energy contribution of principal components.

2) Frequency based similarity matching: We have tried to establish some uniformity between the principal component of PPG and accelerometer signal. Since the principal components are itself the time series, spectrum estimation is exploited where the dominant frequency is utilized as the similarity metric. Formally, we have applied the Fourier Transform for the spectrum estimation and tried to match the dominant frequency of the principal components obtained from PPG and accelerometer signal. The process is defined as

$$\mathbf{FQPPG_{max}} = \underset{\mathbf{k}}{argmax}\, \mathbf{FQSP_{PPG}(k)} \tag{9}$$

$$\mathbf{FQACC_{max}} = \underset{\mathbf{k}}{argmax}\, \mathbf{FQSP_{ACC}(k)} \tag{10}$$

$$\mathbf{d} = \mid \mathbf{FQPPG_{max}} - \mathbf{FQACC_{max}} \mid \tag{11}$$

Where $\mathbf{FQSP_{PPG}(k)}$ and $\mathbf{FQSP_{ACC}(k)}$ are the frequency spectrum of PPG and accelerometer signal respectively.

If the absolute difference $\mathbf{d}$ is less than the threshold then the principal component is subjected for elimination. The threshold is defined as the frequency resolution catered by the Fourier Transform.

Although this method is beneficial, the problem arises when PPG and motion signal coincides and the dominant frequencies of both signal matches. This process would eliminate the principal component associated with true cardiac cycles. In order to avoid this kind of circumstances, along with similarity matching, energy contribution of the particular principal component is also considered. The energy contributed by the PC is also validated with a particular threshold and selected accordingly. The threshold is chosen heuristically.

After discarding, the remaining principal components are considered for the signal reconstruction. It is imperative to note that as we are only computing the heart rate, rather than transforming to the original time domain, the clean signal is reconstructed by only adding the selected principal components.

$$\mathbf{CleanPPG} = \mathbf{PC(i)} + \mathbf{PC(j)} + \mathbf{PC(k)} + ... + \mathbf{PC(t)} \tag{12}$$

Where, i,j,k and t are the arbitrary indexes chosen selectively. To visually understand the effectiveness of our algorithm, we have plotted the frequency spectrums in Figure 2. Figure 2(a) depicts the frequency spectrum of preprocessed PPG signal for a particular time window. Here we can see that the motion peak predominates and the cardiac peak is smaller. Figure 2(b) shows the frequency spectrum of the reconstructed PPG signal after the motion artifact removal, for the same window, where the cardiac peak is clearly visible as the dominant peak

E. Heart Rate Estimation Using Frequency Analysis

Inherently, rhythmic nature of the heart generates a pulsatile component in the arterial blood which manifests a quasi-periodicity in the PPG signal. Essentially, estimation of heart rate is to find the periodicity of the PPG signal of a particular time window and subsequently compute it for a minute. The frequency spectrum is obtained using the Fourier Transform, from which the dominant frequency with maximum amplitude is selected. Instead of using any extensive post-processing method, we have only restricted the search range of the frequency spectrum for the current window. The search range is computed using the previous estimation of the heart rate. The process is defined as follows:

$$\mathbf{FQPPG_{max}} = \underset{\mathbf{k}\in[\mathbf{F_i},\ldots,\mathbf{F_k}]}{argmax}\, \mathbf{PC_{PPG}(k)} \tag{13}$$

$$\mathbf{HR} = \mathbf{FQPPG_{max}} \times \mathbf{60} \tag{14}$$

Where F_i and F_k are the range of the frequencies obtained according to the previous estimation.

IV. EXPERIMENTAL RESULTS

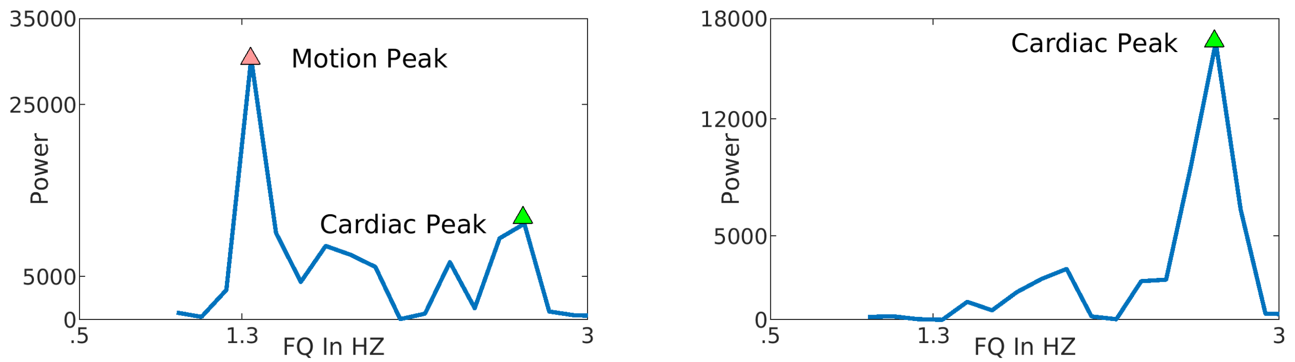
To demonstrate the efficacy of the algorithm we have tested our algorithm with the data set from the 2015 IEEE Signal Processing Cup. The dataset incorporates 12 training and 10 test data set which were accumulated from 18 to 58 years old participants subjected to various physical activities. All sensors data are sampled at the 125Hz sampling rate. The physical activities include walking or running on a treadmill for various intervals and intensive forearm and upper arm exercise. For every participant, two channels of PPG signals, three channels of simultaneous acceleration signals were acquired from a wrist worn device. Additionally, the ECG signal was also obtained simultaneously from the chest using ECG sensors placed at the chest of the participant. The ground-truth heart rate was computed from the ECG signal which is utilized as the evaluation metric for the algorithm's performance. The complete details of the dataset are provided in [1].

A. Performance Metrics

In order to evaluate the performance of our algorithm three performance matrics are used:

- **Average Absolute Error**: The Average Absolute Error is defined as:

$$\mathbf{AvgError_{ABS}} = \frac{\mathbf{1}}{\mathbf{N}} \sum_{\mathbf{i=1}}^{\mathbf{N}} \mid \mathbf{BPM_{Est}(i)} - \mathbf{BPM_{GT}(i)} \mid \tag{15}$$



(a) Frequency Spectrum Of PPG

(b) Frequency Spectrum Of Reconstructed PPG

Fig. 2. The Frequency Spectrum of preprocessed and reconstructed PPG signal respectively

Where $\mathbf{N}$ is the total no of windows. $\mathbf{BPM_{Est}(i)}$ is the estimated HR for the ith time window and $\mathbf{BPM_{GT}(i)}$ is the actual HR.

- **BlandAltman plots:** The BlandAltman plot [3] is the graphical and statistical elucidation of the two measurement methods of the same entity. The resulting graph is a scatter plot where the difference between the two. measurements are plotted against the mean of the two measurements. A horizontal line is drawn around the mean difference and the 95% Limit of Agreement (LOA) is estimated accordingly. The 95% LOA is defined as the average absolute error ± 1.96 standard deviation of the absolute error $(\mu_{Er} \pm \sigma_{Er})$.
- **Pearson correlation:** Pearson correlation is a measure of the linear correlation between Ground truth HR and estimated HR. Higher correlation value manifests better accuracy.

B. Results

To assess the performance of the algorithm, We have tabulated the comparative result of the algorithm in Table I with popular state of the art TROIKA [1] and JOSS [2]. Although the reported methods outperform our algorithm by a little margin, the implementations are quite complex. TROIKA and JOSS exploit the sparse signal recovery (SSR) algorithm for spectrum estimation which is computationally extensive. Conversely, we have used the Fast Fourier Transform(FFT) which is very much feasible in Wearable Device. Moreover, the reported algorithm are heavily dependent on the post-processing process; subsequently integrates several parameters for history tracking and smoothing. Increasing the complexity in post processing hinders the generalization ability. On the contrary, we have only defined and restricted the heart rate range from the previous estimation as a post-processing process.

For further analysis, we have evaluated the performance of the algorithm by utilizing the Bland-Altman plot. The Bland-Altman plot for 12 datasets is illustrated in Figure 3, where the LOA is restricted to [-9, 7.7] BPM.

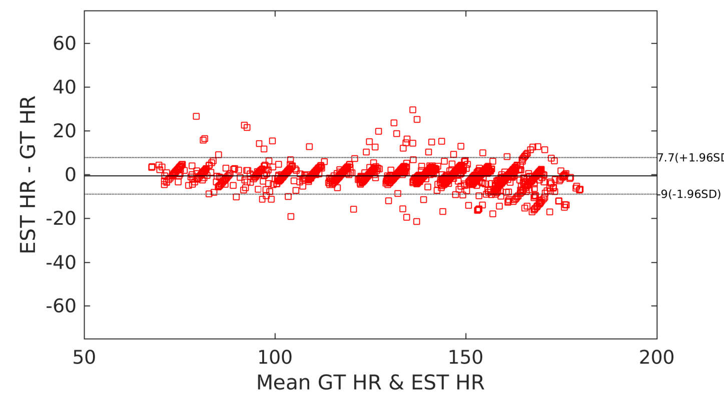


Fig. 3. Bland-Altman plot on the 12 datasets between the ground-truth and the estimates of our proposed algorithm

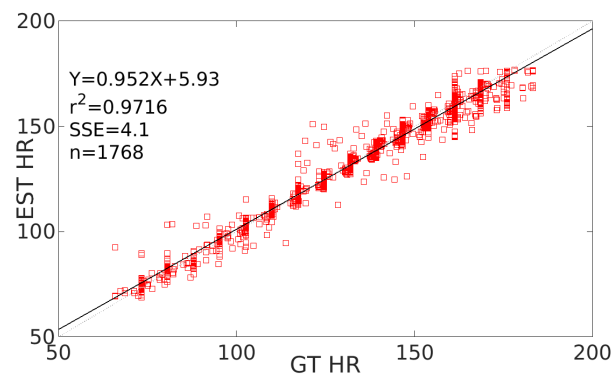


Fig. 4. Scatter plot on the 12 datasets between the ground-truth and the estimated HR of our proposed algorithm

In order to further assess the performance visually, the scatter plot between the ground-truth HR values and the estimated HR is illustrated in Figure 4. The fitted line is represented by $\mathbf{Y} = .\mathbf{952X} + \mathbf{5.93}$ where $\mathbf{X}$ indicates the

TABLE I
HR ESTIMATION RESULT IN TERMS OF AVERAGE ABSOLUE ERROR.

	SUB1	SUB2	SUB3	SUB4	SUB5	SUB6	SUB7	SUB8	SUB9	SUB10	SUB11	SUB12	AVG
TROIKA	2.87	2.75	1.91	2.25	1.69	3.16	1.72	1.83	1.58	4.00	1.96	3.33	2.42
JOSS	1.33	1.75	1.47	1.48	0.69	1.32	0.71	0.56	0.49	3.81	0.78	1.04	1.28
Algo	3.01	2.99	2.04	2.18	2.31	2.53	2.51	2.29	1.93	5.01	3.20	4.20	2.85

ground-truth heart rate, and $\mathbf{Y}$ indicates the corresponding estimated HR. The goodness of fit R^2 is denoted as 0.97.

It is worthwhile to mention that, since in dataset the time window is restricted to 8 seconds, the effective frequency resolution for DFT is limited to 0.125 Hz which imply 7.5 BPM error in heart rate estimation. Considering the frequency resolution for DFT the performance of our algorithm is quite reasonable.

V. CONCLUSION

In this paper, we have proposed a subspace based, real-time algorithm for estimation of Heart Rate from wrist-based PPG signal corrupted by motion artifacts. Our preliminary results show the promise for an efficient method by achieving an accuracy comparable to more complex approaches. As a future path of research, we would like to explore the possibility of reconstruction of motion corrupted PPG signal for more advanced analytics like hypertension and heart-rate variability monitoring from ambulatory PPG.

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